Elastic Stability of Thin-Walled Cylindrical and Conical Shells under Axial Compression

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Results of an extensive experimental program on the stability of cylindrical and conical shells under axial compression are presented and discussed. The experimental data indicate that the buckling coefficient varies with radius-thickness ratio. A study of other data in the literature showed that most of the experimental results fell within or near the scatter-band obtained in the present evaluation. A lower bound design curve is also contained in the paper.

Nomenclature

C = axial compression load coefficient ($P/2\pi Et^2 \cos^2\alpha$ for cones or $P/2\pi Et^2$ for cylinders)
C^* = axial compression buckling coefficient suggested by Kanemitsu and Nojima, including length effect
D = flexural stiffness of shell wall $[Et^3/12(1 - \nu^2)]$
E = Young's modulus of shell wall material
L = axial length of cylinder or cone
l = slant length of cone
P = total axial load at buckling
R = cylinder radius
R_1 = radius of small end of cone
R_2 = radius of large end of cone
t = shell wall thickness
α = semivertex angle of cone
ν = Poisson's ratio of wall material
ρ_1 = radius of curvature at small end of cone
ρ_{av} = average radius of curvature of cone $[(R_1 + R_2)/2 \cos\alpha]$
σ, σ_{cr} = critical average compressive stress $(P_{cr}/2\pi Rt)$

Introduction

THE stability of cylinders under axial compression is a problem that has been studied both theoretically and experimentally by many investigators.[1] Intense interest was initially generated by serious disagreement between experimental data and the results predicted by small deflection theories of buckling, a disagreement not encountered previously with regard to columns and plates. Theoretical investigations of the post-buckling behavior of cylindrical shells[2-4] revealed that the problem differed from the buckling of columns and plates in that neighboring equilibrium states were unstable, i.e., deformations could occur with a decrease in applied load. It was also found that the load-carrying capacity of cylinders was extremely sensitive to initial imperfections of the order of a fraction of the wall thickness, which fact has since found acceptance as an explanation of the discrepancy between theory and experiment. More recently, additional factors, such as nonuniformity of loading around the shell circumference[5] and nuclei of plastic strain,[6] have been suggested to explain the large amount of scatter in test results. The original purpose of the investigation was to consider the load carrying capacity of conical shells alone. Since the scatter of the axial compression results of other investigations was so large for cylinders, however, it was decided that a pilot program of cylinder tests would be necessary to yield comparable data as a check on the cone results and as a basis for obtaining a design criterion for conical shells postulated on the existence of an equivalent cylinder. The cylinder investigation soon outgrew the pilot program stage, however, as many factors not mentioned before in the literature intruded and demanded study. The net results of these studies, which attempt to bring some order to the study of cylindrical shells in compression, also manage to inject more unknown factors in the problem.

The controversy surrounding the design of cylindrical shells has not yet been fully extended to conical shells, since serious study of these structures began only a few years ago. In the present investigation, the results of the axial compression test program carried out for conical shells of various geometries are given and recommendations are made for their design. It is obvious that, as the number of conical shell tests grows, the controversial aspects of design may exceed those associated with cylinders, if only because the conical shape permits another degree of freedom to the variables to be considered.

Experimental Technique

The specimens used in the experiments were made of Mylar polyester sheet. This has been found to have a Young's modulus of 700,000 psi, a Poisson's ratio of 0.3, and proportional limit and yield stresses of 6000 and 11,000 psi, respectively. Because of the small variation of the properties of Mylar from roll to roll, the modulus of elasticity was determined for each specimen by using a load-deflection curve.

Specimens were made by cutting accurately developed cones and cylinders from the Mylar sheet, allowing $\frac{1}{2}$ in. on the top and a minimum of $\frac{3}{4}$ in. on the bottom for clamping and $\frac{3}{4}$-in. overlap for the longitudinal seam. All the cones were made with the same base radius to reduce the number of base clamping fixtures needed. The developed cone was then wrapped firmly about a conical wooden mandrel. A lap joint fastened with double-backed adhesive cellophane tape was used for the longitudinal seam. In later tests, the bonding material was changed to Eccobond, an epoxy cement, after

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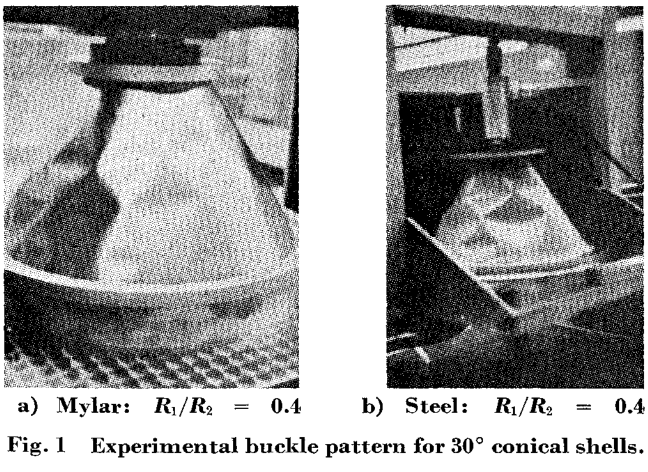
experimentation with various bonding agents. Although the thin Mylar sheet is quite flexible, the use of carefully laid out patterns and conical assembly mandrels made it possible to obtain specimens that were dimensionally accurate and relatively free of initial wrinkles or bulges. Steel specimens, seam welded and spun, were obtained from an outside source.

The Mylar cones and cylinders were assembled for testing in clamping fixtures with the bottom clamp resting on the base of the loading device. A circular plate with a hole in its center was then centrally placed on the cone with a steel ball resting in the hole. The load cell attached to the loading screw was then placed on top of the steel ball, and the cone shifted until the load cell was vertical. The assembled specimen was then loaded in axial compression by turning the load screw at a relatively constant rate. The load was increased until failure occurred (see Fig. 1). Failure of the specimen was usually quite sudden, with diamond shape buckles snapping into position. In later tests, the specimens were clamped in aluminum end-plates filled with cerrobond, a low melting point alloy.

On some tests a small dimple appeared next to the seam and grew as the load was increased until the cone collapsed at a very low load. The addition of scotch tape to the seam or an increase in the seam width eliminated this type of failure and, consequently, increased the buckling load. After each failure, the buckle dimensions, buckling load, and the extent of the buckles around the circumference of the cone were recorded. The Mylar specimens have the ability to recover completely after being buckled, presumably because the cone remains elastic in the postbuckling state. Therefore, it was possible to perform several tests on one cone.

The first axial compression test on every cone was done with the steel ball and loading plate centrally located. At buckling, the location of the first buckle was recorded. In the second compression test of the cone, the loading plate and steel ball was moved, on a line with the center of the cone, away from where the first buckle appeared. This procedure was continued until a maximum compressive load was obtained. It was felt that this procedure eliminated most of the effects of eccentricity of loading.

Steel specimens were fixed in Cerromatrix in the upper and lower clamps and then placed in the Baldwin Universal testing machine (Fig. 1). A swivel top plate was placed in the Baldwin test machine so the top clamping plate and the loading plate could align themselves. A compressive load was then exerted on the specimen until buckling occurred. Failure was usually quite sudden with diamond shape buckles snapping into position. Premature dimpling due to imperfect seams was eliminated by the addition of a $1\frac{1}{2}$-in. shim strap attached with an epoxy cement along the seam. After failure, the buckling load and buckle sizes were recorded. When the load was released, the buckles disappeared from the steel specimens with large R/t values. These specimens



a) Mylar: R_1/R_2 = 0.4 **b) Steel: R_1/R_2 = 0.4**

Fig. 1 Experimental buckle pattern for 30° conical shells.

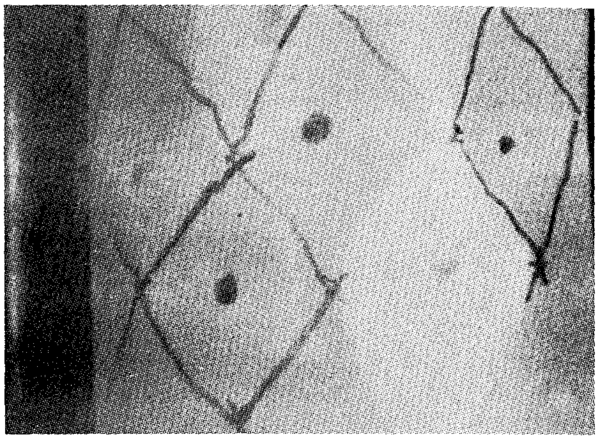


Fig. 2 Illustration of shift of buckle pattern for retested Mylar cylinder (dots show location of buckle corners in previous test).

were examined closely, and no evidence of plastic deformation was noticed. When the specimens were retested, however, the second buckling load was always much lower than the first. A qualitative explanation of this phenomenon is given in Ref. 6. During the initial compression test on the shell, failure occurred at some stress level and was initiated at that stress level by some initial imperfection of the shell. The failure took the form of a diamond shaped buckle pattern around the circumference of the shell. The shell supported a load after failure, and this load was carried in the crests of the buckles with the points of highest stress occurring at the junction of adjacent buckles. At these points the stress was above the yield stress, and microscopic plastic deformation occurred. Evidently these microscopic plastic deformations have a greater effect than the initial imperfections that were built into the shell since, in the next test, failure was initiated at these points of highest stress concentration at a lower stress level, with the points of maximum deflection of the new diamond pattern at the same location as the node points of the first.

Figure 2 shows the buckle pattern of the second test for a Mylar cylinder. The dots at the center of the diamonds mark the location of previous node points. The new node points are marked with an X. It can be seen that the node points of the first test coincide with the centers of the new set of buckles. This behavior occurs in metal specimens as well as Mylar specimens. For Mylar, however, the degree of plastic deformation is less than for metals, and the buckling load is not affected.

Results

Cylinders

The experimental data obtained during the course of the present program are listed in Ref. 27. These data have been collected from the results for axial compression alone and from the endpoints obtained in other investigations of axial compression combined with internal pressure, external pressure, or bending. They are therefore associated with a multiplicity of loading devices, end fixtures, and testing techniques. The results are plotted in Fig. 3 (the circles) in the form of values of the buckling coefficient $P/2\pi Et^2$ or C as a function of radius-thickness ratio R/t. Results for $L/R <$ 0.5 were not plotted.

The results indicate a trend similar to that obtained by other investigators in that the buckling coefficient appears to vary with radius-thickness ratio. A study of the data in the literature[7–22] indicates most of the experimental results fell within or near the scatterband obtained in the present program and, hence, that a discriminating choice of data might permit some conclusions to be drawn for design purposes.

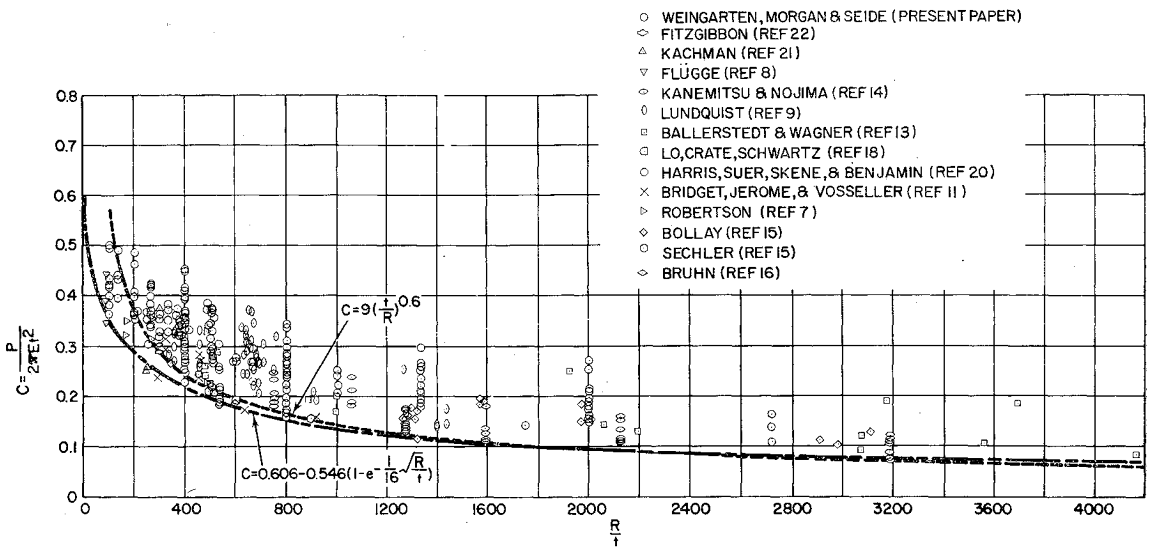


Fig. 3 Comparison of various experimental results for cylinders in axial compression.

The choice of data was guided by several considerations. First of these was the conclusion that the effect of length could not be readily discerned if the length-radius ratio was within the range from 0.5 to approximately 5, for which most of the data applies. A length effect exists for cylinders with values of L/R less than 0.5. For values of L/R greater than 5, the few data points available indicate that there *might* be a length effect, but there are insufficient data to establish any trends. Investigations yielding buckling coefficient consistently lower than the present results were omitted entirely on the premise that fabrication methods and testing techniques were significantly inferior. Those test results in which the critical stress was greater than about 70% of the yield stress were likewise omitted. The resulting data are also plotted in Fig. 3.

The entire group of data is reasonably consistent, despite the large amount of scatter. The relation suggested by Kanemitsu and Nojima for long cylinders

$$C = 9(t/R)^{0.6} \tag{1}$$

can be seen to be a good lower bound to the data for the range R/t greater than 500. For lower values of R/t, the equation becomes increasingly unconservative. An alternate relation that appears to give a good lower bound over the entire range of R/t tested is also shown and is represented by the equation

$$C = 0.606 - 0.546\,\{1 - \exp[-\tfrac{1}{16}\,(R/t)^{1/2}]\} \tag{2}$$

The data obtained thus far indicate that the radius-thickness ratio is a significant parameter indicative of the trend of the results. The scatter associated with any particular value of radius-thickness ratio shows, however, that the radius-thickness ratio representation disguises important factors. It has been suggested that the scatter can be explained by assuming different magnitudes of initial imperfections for various specimens. This explanation serves for the scatter obtained in the present program for ostensibly identical specimens made by a single individual and tested under similar conditions, since the initial imperfections would be expected to occur with statistical distribution. However, observations made in the course of testing indicate that the statistical sample can be biased.

It was observed, for instance, that many of the lower points of the scatterband are associated with earlier phases of the test program and that test results obtained at a considerable later date would yield a higher mean level. Thus the experience and competence of the fabricator and experimentalist should be taken into consideration. Another factor that evidently influences the results is the wall stiffness or size of the specimen, in that specimens attempted with Mylar gages of 0.001 and 0.002 in. were exceedingly hard to handle without introducing wrinkles. Additional factors were observed as well. In many of the tests of the present program, the potting material used was the low melting point alloy Cerrobend, in others, Cerrolow was used. The main differences between these alloys are their thermal expansion characteristics. Cerrobend expands while cooling, whereas Cerrolow contracts a much smaller amount while cooling. It was found that those tests conducted with Cerrolow gave results that were consistently higher than those obtained with Cerrobend. One would assume, then, that the bulging of the ends of the cylinder walls is an important parameter. Finally, it was also observed, both in the present tests and from a study of the data in the literature, that results obtained when the nominal stress level was on the order of 70% of the yield stress or greater yielded consistently low buckling coefficients. This effect can also be traced to end conditions, since it can be shown that yielding of the restrained ends due to bending would start at about this level. With all of these factors influencing the results, it is interesting to note, however, that the lower bound of the various test results is reasonably consistent, indicating that there is a practical lower limit to the combined effects of all of the parameters influencing the results.

The results in Ref. 27 indicate that the buckling coefficient increases for small values of L/R. A plot of this data for $L/R < 1$ is given in Fig. 4 for the R/t values of 100, 200, 400, and 800. The data are reasonably consistent with an increase in the scatter as L/R decreases. The results do not show any appreciable change in the buckling coefficient until $L/R < 0.5$. Within this range a correction is needed for the length effect. The relation given by Kanemitsu and Nojima for short cylinders

$$C^* = 9(t/R)^{0.6} + 0.16\,(R/L)^{1.3}\,(t/R)^{0.3} \tag{3}$$

can be seen to be *unconservative* for R/t less than 800. A relationship that appears to give a good lower bound for all R/t and L/R tested is

$$C = 0.606 - 0.546\,\{1 + \exp[-\tfrac{1}{16}\,(R/t)^{1/2}]\} + 0.9\,(R/L)^2\,(t/R) \tag{4}$$

The length effect in Eq. (4) is obtained from the classical simply supported flat-plate relationship. A change of the buckle pattern with length for an R/t equal to 800 is shown in

Fig. 5. For L/R equal to 1 and $\frac{1}{2}$, the diamond buckle pattern does not change shape and the length effect should be negligible. For $L/R < 0.5$, the buckling pattern changes into an apparent higher energy configuration. The axisymmetric mode appears at L/R equal to $\frac{1}{8}$. The axisymmetric type shape closely approximates the buckle shape of a plate of infinite width. Therefore, one should not expect a change in buckling coefficient until $L/R < 0.5$. At this point, the buckling coefficient should increase, approaching the critical value of an infinite width simply supported flat plate as $L \to 0$. The length effect given in Eq. (4) attempts to follow this reasoning.

Conical Shells

The theoretical analysis of Ref. 23 yields the critical axial load coefficient for conical shells as a modified form of the result for cylinders, namely,

$$C = \frac{P}{2\pi E t^2 \cos^2\alpha} = \frac{1}{[3(1 - \nu^2)]^{1/2}} \tag{5}$$

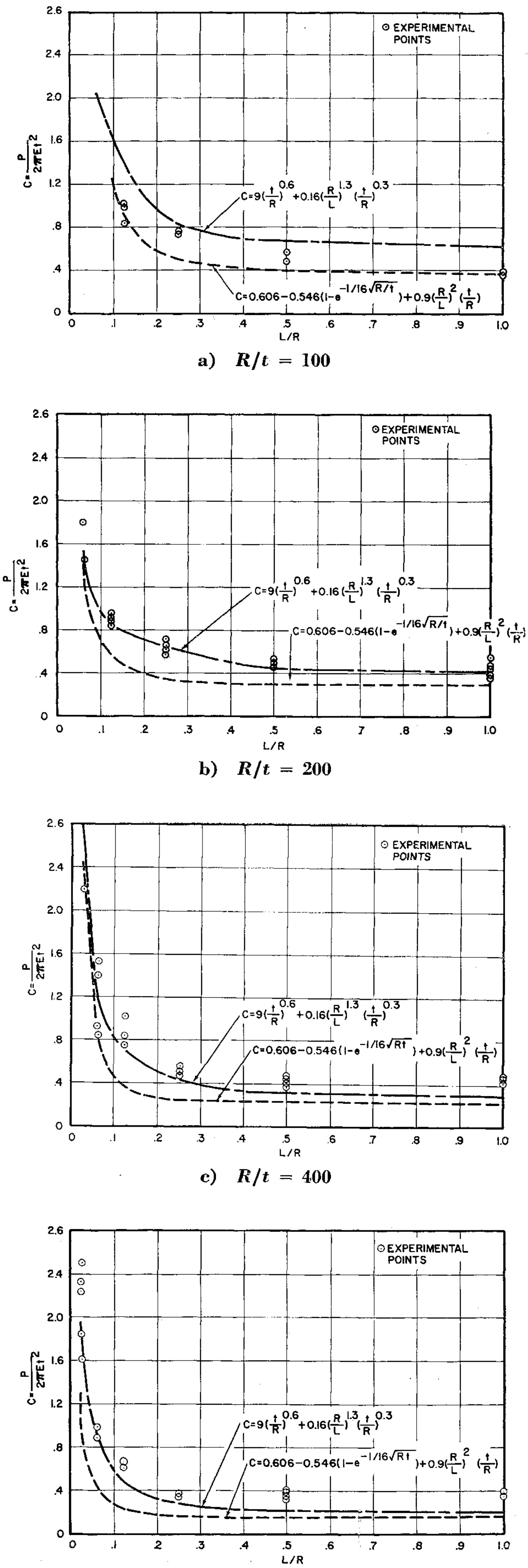
It was also suggested in Ref. 23 that the buckling load coefficient C for conical shells might be similar to that for an equivalent cylinder having the same wall thickness, a length equal to the slant length of the cone, and a radius equal to the average radius of curvature of the cone. The specimens were designed to investigate this hypothesis.

The results of the various tests are given in Ref. 27 in the form of values of $P/2\pi E t^2 \cos^2\alpha$. In several preliminary analyses of the data (Refs. 24 and 25), attempts were made to verify the foregoing hypothesis, with inconclusive results. The data for cones appeared to be consistently higher than those for cylinders. It has since been determined that, if the equivalent cylinder is assumed to have a radius equal to the *small* radius of curvature of the cone, much better agreement is obtained. This conclusion was reached by a comparison of the results for both the cones and the cylinders with results predicted by the Kanemitsu-Nojima equation for C^*, Eq. (3). A comparison of the values of C/C^* for cylinders and cones, given in Ref. 27, indicates that the scatterband for both is similar. The experimental values for cones are compared with the lower bound curve for cylinders in Fig. 6, where it can be seen that the agreement is fair. It should be noted that many of the high points correspond to cones having length/small radius of curvature ratios less than 0.5.

It is surprising that the small radius of curvatures should be significant, rather than the average radius of curvatures, since buckle patterns of conical shells (see Fig. 1) show that buckles do not particularly predominate near the top of the specimen. The answer to this problem may lie in an examination of the large deflections of conical shells, which is unavailable at present. The findings of the present paper contradict those of Ref. 26, where the *large* radius of curvature was recommended for the equivalent cylinder. However, the edge conditions of the tests reported therein were such as to render the results invalid for conical shells attached to end plates or stiffened by end rings.

Summary and Recommendations

Although the monocoque circular cylinder under a uniform axial load is one of the simplest of shell configurations and is one that has been studied by many investigators, a completely satisfactory solution to the problem is not yet available. It has been found that many factors combine to determine the buckling load of a particular cylinder. These include the care in fabrication, the patience and experience of the investigator, end conditions of the test specimen, initial deformations, plastic strain nuclei, etc. Fortunately, it appears that there exists a fairly definite lower bound to the buckling stress for cylinders that buckle at stresses in the



a) $R/t = 100$

b) $R/t = 200$

c) $R/t = 400$

d) $R/t = 800$

Fig. 4 Effect of length/radius ratio on buckling coefficient.

elastic region. This lower bound can therefore be used for design purposes with a reasonable assurance that the structure will be safe. A discriminating comparison of available data indicates that the lower bound for the variation of the buckling load with the radius-thickness ratio of the circular cylinder is reasonably well described by the relation

$$P/2\pi Et^2 = 9(t/R)^{0.6} \tag{6}$$

for R/t greater than 500 and for L/R's less than approximately 5 but greater than 0.5. A relation that yields a better representation of the lower bound curve for the entire range of test data,

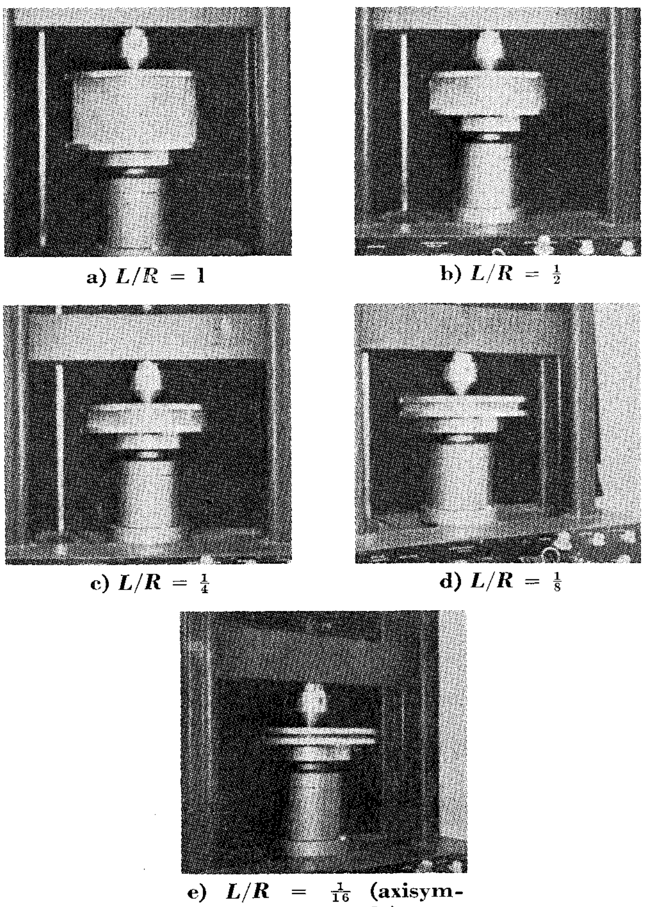
$$100 < (R/t) < 4000 \qquad 0.031 < L/R < 5 \tag{7}$$

has been found to be given by

$$\frac{P}{2\pi Et^2} = 0.606 - 0.546\left\{1 - \exp\left[-\frac{1}{16}\left(\frac{R}{t}\right)^{1/2}\right]\right\} + 0.9\left(\frac{R}{L}\right)^2\left(\frac{t}{R}\right) \tag{8}$$

For design purposes, then, the situation is as follows: There appears to be no real reason to radically depart from past design methods provided that the range of cylinder parameters does not exceed those tested. The basic Kanemitsu-Nojima formula, giving the radius-thickness ratio effect, might be replaced by Eq. (8), which includes the L/R effect and would extend the lower limit of applicability of these design methods.

In order to obtain a more accurate design curve, it may be necessary to undertake an extensive experimental program, under relatively constant conditions, to determine a statistical design criterion that better describes the effects of radius-thickness ratio and length-radius ratio. This program should involve many identical specimens to give a reasonable statistical sample for each combination of values of L/R and R/t. Care should also be taken to eliminate the possibilities of yielding at the cylinder ends or over-all plastic buckling and the separate determinations of these effects made with many different materials. Future theoretical work should include a better large-deflection analysis, including the effects of finite length, end conditions, and plasticity.



a) $L/R = 1$ b) $L/R = \frac{1}{2}$ c) $L/R = \frac{1}{4}$ d) $L/R = \frac{1}{8}$ e) $L/R = \frac{1}{16}$ (axisymmetric mode)

Fig. 5 Change of buckle pattern with length (R/t = 800, t = 0.005).

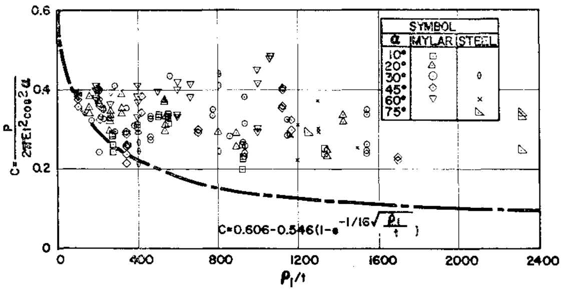


Fig. 6 Comparison of axial compression coefficients for conical shells with lower bound curve for cylinders.

For conical shells in axial compression the same arguments apply. The data of the present report indicate that for design purposes it is adequate to modify Eqs. (1) or (7), and (8) in accordance with the theoretical result of Ref. 23, and, by use of the small radius of curvature and the slant length of the cone in place of the cylinder radius and length, to obtain

$$\frac{P}{2\,\mathrm{E}t^2\cos^2\alpha} = 0.606 - 0.546\left\{1 - \exp\left[-\frac{1}{16}\left(\frac{\rho_1}{t}\right)^{1/2}\right]\right\} + 0.9\left(\frac{\rho_1}{l}\right)^2\left(\frac{t}{\rho_1}\right) \tag{9}$$

The loads calculated by this formula bear about the same relation to experimental loads for cones as for cylinders. It would be advisable however to investigate the effects of the various geometric parameters in much more detail in order to determine differences in behavior not brought out by the present set of experiments. In this respect it would be of great interest to have large-deflection studies of the post-buckling behavior, including initial imperfections, to guide the experiments if any great difference in behavior actually exists.

As a result of this investigation, a number of concepts concerning cylindrical and conical shell buckling have been clarified, and some new problems requiring further investigation have been uncovered. As in many complex problems, a satisfactory solution depends upon a close association of careful experimentation and sound theoretical studies. Theory is useful in establishing the significant parameters, and experiment is required to check the accuracy of the theoretical approach and to point out phenomena that have either been ignored or not sufficiently well considered in the analytical approach. The nature of the shell buckling problem seems to be such that, although the theory correctly predicts the important parameters involved, design buckling loads must be found by experimental methods, since the current theories are not sufficient to determine exact numerical values of the parametric coefficients.

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